

AN INTEGRATED TOOL FOR COMPETITION GO-KART TRACK ANALYSIS

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ABSTRACT – Competition go-karts must satisfy the philosophy of extreme simplicity, but their design can be even more complex than the one of standard vehicles. Karts until today have been developed mainly on the basis of experimental tests and personal experience of skilled technicians. However, the increasing technological content of these vehicles nowadays requires design and analysis criteria that are peculiar to modern engineering. Even if experimental tests are still the most common and extensively used approach, the numerical simulations has been developed so much in the latest years to become rapid and reliable. On one hand, this allows to support the experimentation, reducing the number of experimental test to be done; on the other, it produces a complete description of the physical quantities involved, permitting a deep understanding of real phenomena and behaviour. The success of a competition vehicle depends on several factors including vehicle dynamics, engine performances, and aerodynamic effects.

In this paper an integrated tool, addressed to lap time simulation of a competition go-kart, is presented. All the mentioned components of vehicle dynamics are taken into account. The aerodynamic performance of a go-kart is predicted by means of a CFD software that solves indirectly the equations for the continuum (Navier-Stokes) by reproducing the fluid dynamics at the molecular level. It has to be noted that aerodynamics effects act not only on the drag and lift forces but also on the engine heat transfer capability and on the dynamic load transfer. Subsequently the engine performance can be evaluated using a mono-dimensional fluid dynamic code that is able to simulate the whole engine, from air box to exhaust system, taking in account the most important engine parameters. Vehicle longitudinal and lateral dynamics was computed by means of a solver dedicated to competition go-kart dynamic analysis that is able to foresee the vehicle behaviour responding to driver commands. The numerical model was tuned in order to match a series of acquired track results.

A general purpose track model was then developed suitable for the representation of testing track or race circuits and able to manage an arbitrary path lying within its boundaries.

It is then possible to simulate an arbitrary manoeuvre, collecting aerodynamic, engine and vehicle models results into the main simulation program, and taking advantage of the track model. Special manoeuvres like steering pad, double change of line and spiral trajectory and whole circuit lap have been considered. The entire tool is sensitive to aerodynamic, handling, engine and tires parameters and then is useful to find optimised design and set-up solutions. A practical application is presented about the global tuning of a go kart to minimise the lap time in an actual race track.

INTRODUCTION – Lap time simulation is a very fascinating topic, but to become an useful race engineering tool needs several disciplines to be involved. In fact the overall performance of a racing car, that can be well quantified by the lap time, is the result of the interaction between a lot of design and set-up parameters and, of course, of the ability of the driver to squeeze the characteristics of its car to the limit. A lot of research work was devoted to this matter even if only a little part emerges in the published literature for obvious secrecy reasons. An interesting analysis was recently presented for the race optimisation of AUDI car in Le Mans 24 hours competition [1]. A systematic approach for race car optimisation is widely discussed in [2]. Contributions strictly addressed to lap time analysis for competition go kart are still not published in technical literature. A lot of works are however available about several topics that concur in lap time analysis.

As far as vehicle dynamic is concerned, simulation models have been extensively developed for standard vehicle handling and stability analysis [2, 3]. Nevertheless the prediction of the go-kart lateral dynamics requires to develop specific models, because of the absence of differential gear and suspension systems and an appropriate approach has to be employed in simulations instead of classical rigid bodies technique. In fact, dealing with a so particular vehicle, the elastic behaviour of the chassis has a great influence on road dynamics,.

Several works have been presented on kart structural aspects and track dynamics. Much of theme regard the structural aspect of the frame, like [4, 5]. Others consider the whole vehicle behaviour by an means of experimental test [6] or simulating the kart behaviour with appropriate models by means of commercial multi-body software [7] or adapting the single-track model to kart peculiarities [8, 9].

Also the authors have studied the problem of kart dynamics, by means of appropriate modification of rigid body approach. The work described in paper [10] is an investigation on the powerful and usability of multi-body environment to study kart behaviour on track. In paper [11], the effect of driver mass has been examined with multi-body technique, in order to highlight the influence of driver position and motion on the whole vehicle dynamics. Furthermore a Fortran code, able to simulate kart behaviours in road manoeuvres, has been employed to investigate lateral dynamics, taking into account the effect of different frame geometry configurations [12]. Tyre characteristics have been tuned by means of experimental measures in track sessions.

The study of the whole kart dynamics also requires the knowledge of all the contributions to longitudinal dynamics too, i.e. engine force, and aerodynamics resistance. Engine simulation is a widely explored research field and a complete panoramic about set-up and design of two stroke engines can be found in many books such as [13]. Several contributions were also given about simulation of complete motors; the authors themselves have worked in this field about integrated simulation of a complete engine [14] and recently about the design and optimisation of a reed valve [15].

Considering that new kart competition regulations allow almost arbitrary shapes for the bumpers and other components, aerodynamics is going to play a primary role on new kart design programs. Even if wind tunnel measurements continue to be the most common and extensively used approach, with currently available computational equipment the turn around for numerical simulations is becoming so rapid that is feasible to examine an extremely large number of variations. The employment of numeric approach allows to support the experimentation and produces a complete description of the three-dimensional flow field in terms of velocity, pressure and temperature distribution, including other related physical quantities. The importance of an aerodynamic analysis resides not only in predicting drag and lift forces, but also in its influence on vehicle dynamics and engine performance. A detailed study on this topic was presented by the authors in [16] where, for the first time, the aerodynamic behaviour of a competition go-kart was analysed and simulated.

Of course a complete lap time simulation tool requires a virtual driver able to act on steering throttle and brake in order to maximize the speed in every part of the circuit. A review of various literature approach and a model of automated driver has been presented by Gordon et al. [17] that proposed an algorithm able to follow a planned trajectory.

Collecting the aforementioned experiences of the authors an integrated tool for the simulation of competition go kart was developed and is herein presented.

SIMULATION TOOL - Several building blocks were prepared and individually checked and optimised to be suitable for competition go kart simulation: vehicle model, engine model, aerodynamic model, track model. The simulation tool developed consists in a software, developed in Windows environment, capable to manage the calculation activities of each block.

Engines parameters were used to launch a one-dimensional gas dynamic simulation that produces as result the working map of the engine in a format suitable for longitudinal vehicle dynamic analysis. However the use of an experimental map in the same format, if available, could be inserted.

In the same way, drag data for various configuration were computed and transferred to the lap time simulation program, since CFD simulations require a lot of CPU time. Of course the same data could be derived experimentally.

Vehicle model interacts with the program at two levels. The first approach consists in a preliminary construction of stationary g-g diagrams related to the current vehicle setup; such data are then used by simple dynamic solver based on point mass vehicle model. Finer results could be achieved controlling the full vehicle simulator along the entire lap. This approach requires a virtual pilot to follow the desired path. As far as the vehicle simulator is concerned, two solvers are available: an internal developed simulation program, and a commercial multi-body model (Working model 3D).

At this stage of development only the first approach is fully working, for this reason the problem of automated driver will not be pursued in this paper.

Simulation core is very simple. Firstly global input parameters (engine, vehicle) are processed to obtain engine map and vehicle g-g diagram. Secondly the trajectory is defined by a set of control points. Speed histories are then calculated imposing a braking and driving law that maintains the acceleration vector always on the vehicle admissible acceleration diagram boundary. Lap time is then calculated by simple integration. To speed up simulation time, a set of vehicle and engine set-up can be stored, permitting to see in real time the effect of parameters changes for a given trajectory. Furthermore at fixed engine and vehicle set-up, the best driving could be found varying trajectory shape parameters. Current trajectory is painted in the graphic view port, highlighting the braking point, partial times and others interesting quantities. Furthermore a real time animation of the lap is available: actual perspective of the pilot is shown, together with a series of configurable instruments that show in real time the evolution of computed quantities.

VEHICLE MODEL – The study of kart dynamics needs the development of specific simulation tools. In fact, the absence of suspensions and differential gear requires to modify the typical rigid body approach usually employed in vehicle behaviour simulation. In order to reach good results in applying simulation tools, two tasks must be faced. Firstly, the need of manage the fixed connection between the rear wheels, that impose equal speed for both sides tyres and hence the presence of slipping during cornering. Secondly, the necessity of correctly represent the elastic behaviour of the whole chassis, permitting to exactly evaluate the load transfers and, consequently, the tyre forces. Regarding this second issue several solution have been carried out by the authors. Papers [10, 11, 12] are based on the condensation of the whole chassis stiffness into a 4 by 4 matrix. Extracting, by means of a FE model of the kart

frame, the stiffness matrix associated to the dof's representative of vertical displacements of the wheel hubs, is possible to use the stiffness matrix applying such a similitude with conventional suspension system taking nevertheless into account the coupling between the dof's considered. This approach has been employed both with a model realized in a commercial multi-body environment and with Fortran simulation code, named Kart 2D.

Further developments have conducted to a larger employment of finite element method to evaluate load transfers. Applying mass loads (accelerations in the three direction x, y, z) to a kart model, inclusive of non structural and driver masses, is possible to carry out a set of maps that permit to assess the vertical load acting on each tyre in whatever driving condition (braking, cornering, accelerating). Furthermore the effect of front wheel lift due to steering cinematic has been considered dealing with it like a load condition. Because of the interest to three wheels equilibrium condition (due to the typical rear wheel lift in cornering manoeuvres) both three and four points constraint situations have been examined.

Gathering the results of the fem analyses for every single load case (1 g of longitudinal acceleration, 1 g of lateral acceleration, 1 g of vertical acceleration, 1 mm of front left wheel vertical displacement, 1 mm of front right wheel vertical displacement) into matrix, and applying the principle of superposition, is possible to assess the vertical load on each tyre. The algorithm also provide for switching between 4 or 3 wheels solution, depending by the presence of a negative vertical load on one of the rear wheels.

Once vertical loads on each tyre are known, is possible to evaluate longitudinal and lateral tyre forces. A simplified bi-linear dependence between traction/braking force and longitudinal slip has been employed, basing on literature values to set the model. The longitudinal slip is assessed integrating the equation of motion of the rear axle and comparing the peripheral speed of the contact point, seen as a point in rigid motion with the axle, with the velocity of the vehicle, computed taking into account longitudinal speed and yaw rate. The engine torque computed by the engine model is inserted in the code and further also the aerodynamic and rolling resistance are considered. Therefore is possible to simulate the actual ability of the kart to accelerate.

On the other side lateral behaviour of the tyre has been identified by means of track acquisition. The Fortran model has been employed in a set of parametric runs, permitting to carry out the characteristic of lateral force versus tyre lateral slip. The simplified formula adopted is the following

$$F_{yi} = \frac{F_{f \max}}{2} \frac{F_z}{F_z^*} \left(1 - e^{-\frac{a_f F_z^*}{a_{f \max} F_z}} \right)$$

which presents an almost linear behaviour nearby the zero and a saturation at high values of slip angle. The dependence by vertical load has been estimated basing on the knowledge of static loads and literature values.

Combining all the loads present, the equilibrium is computed by time-step integration of the in-plane equations of motion.

Kart 2D by itself permits to execute whatever analyses by means of the appropriate input files. Simulation of experimental manoeuvres, plots of understeer diagram, handling diagram or g-g diagram as well as parametric analyses can be performed. In fact, in this work, the actual capability of reach combination of longitudinal and lateral acceleration, have been assessed with maps obtained by the outputs of Kart 2D.

ENGINE MODEL – A full gas-dynamic model was implemented in order to calculate the effect of setup parameters on engine performance. One-dimensional non-stationary flow

conditions were handled by a total variation diminishing solver previously developed [14]. Shocks and thermal discontinuity, typical of exhaust flow of two strokes engines, are accurately and robustly resolved using the classical two steps Lax-Wendroff solver improved with a flux correction scheme. Open system conservation equations were used to follow the evolution of thermodynamic variables inside the cylinder, the crankcase and the airbox, considering perfect mixing condition. Furthermore a simple combustion model, represented by a Wiebe function, has been included.

Typical engine layout is shown in figure 1 in which the airbox is represented by a reservoir connected to the external ambient by means of an orifice.

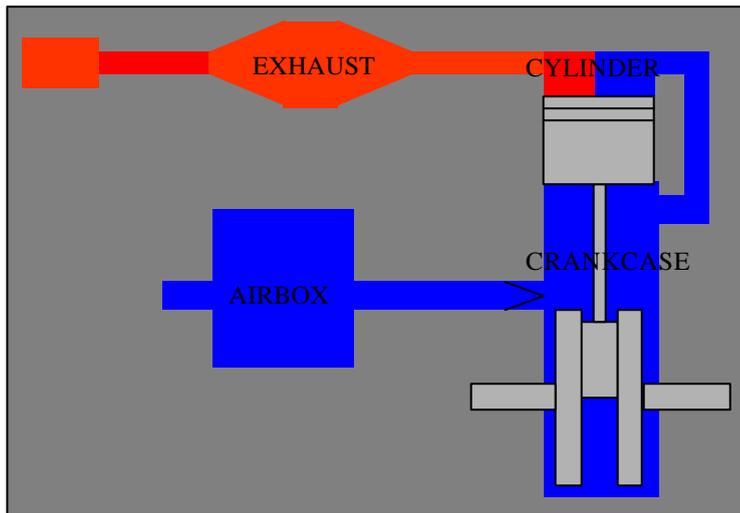


Figure1. Two stroke engine layout.

The duct between the airbox and the crankcase was modelled by means of a pipe element that has with a fixed area orifice at the first boundary and a variable area orifice at the second boundary. Proper values of these areas are imposed adopting a fixed angular base tabled values for piston ported or rotary disc valve engine, or adopting a dynamic model for reed valve ported engines [15].

Crank case and cylinder were connected with a pipe that has a fixed boundary condition at crankcase side and a variable area connection imposed adopting actual section offered by ports valves. Exhaust line was modelled in a similar manner adopting a one-dimensional element, with various cross sections typical of a tuned two strokes engine, connected at the cylinder side by a variable section orifice representative of the port exhaust valve and at the other side to the external ambient.

This simple model is suitable to foresee parameters effects on performance, but need to be tuned with some experimental data to give quantitative correct results.

A lot of parameters can be handled by the models. The most interesting for competition kart setup are the following: ignition angle, reed valve material, exhaust tailpipe length, air/fuel ratio and engine timing that is imposed by the shape of piston ports and by reed valve geometry.

Simulation time are quite time consuming if a coupled simulation with the vehicle is performed solving gas dynamic of each engine revolution. For this reason engine performance data were first calculated for desired parameters set, performing a series of simulation at various RPM and transferring torque data in lookup tables, quickly handled by overall simulation model.

AERODYNAMIC MODEL –The aerodynamic performance was predicted by means of a CFD software package, that has been already successfully applied to motorcycle and automobile aerodynamics.

A complete three-dimensional model of go-kart and driver was developed for CFD analysis (see figure 2). Then this model became a part of a virtual wind tunnel where particular boundary conditions were applied to take into account the sliding of the ground, the rotation of wheels, axle, brake-disc and transmission gear and also the porosity of the radiator.

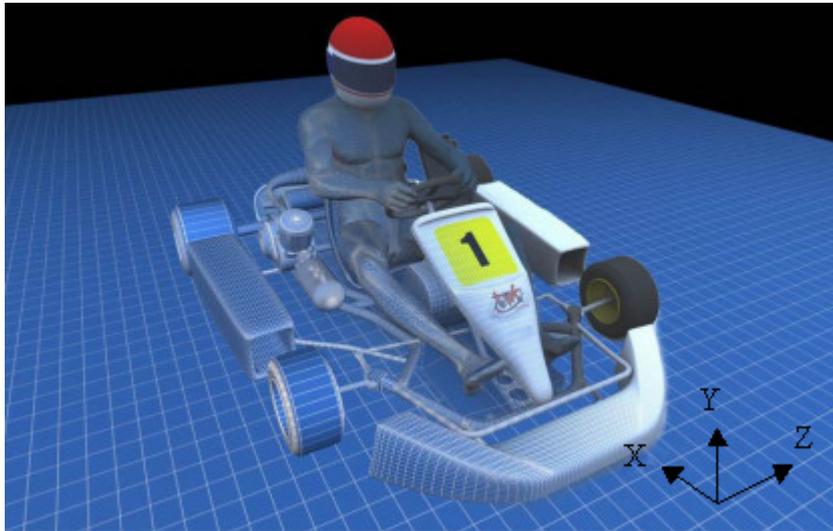


Figure 2. Complete 3D model of the kart

Realised model has been employed to evaluate how aerodynamics influences fluid dynamics conditions of intake and exhaust engine systems. Moreover the aerodynamic effect on the flow rate through the cooling radiator has been estimated in order to assess the heat rate deducted from the engine.

The knowledge of aerodynamics coefficients, obtained with a dedicated CFD simulation for few configurations, is completely descriptive of the aerodynamic behaviour of the kart. In fact the aerodynamic coefficients are not dependent by velocity and forces and moments could be calculated by this coefficient and the velocity in the entire velocity range. Nevertheless the aerodynamics effects on the engine vary with velocity; thus it is feasible to simulate these effects for few velocities and then extrapolate the results for others velocities.

These information have been introduced, in map format, in the integrated simulation tool. Hence the lap time, as far as the whole race time, are calculated considering aerodynamics forces and their effects on vehicle handling and stability, engine performance.

TRACK AND TRAJECTORY MODELS – A generic two-dimensional track model was implemented in order to easily manage typical testing circuit, as steering pad circle or double line change path, as well as every circuit of a competition season.

A very simple mathematical model was chosen to represent the track consisting in an arbitrary number of constant radius arcs or straight segments. For each segment is defined the length, the curvature radius with sign, the width at the end points and a label with the name of the segment. Once the origin of the path is defined, and the starting direction and starting width of the track are chosen, overall circuits can be constructed connecting each segment to the end of the previous, preserving the same tangent, and using the sign of radius to decide the turning direction.

Adopting this simple data structure is possible to define the position, the tangent and the normal vectors of the circuit centreline as a function of curvilinear abscissa.

Left and right boundary are easily represented considering the actual track width, performing a linear interpolation along the abscissa between the start and the end of each segment. Track mathematical model is used for several purposes. The first is obviously the graphic visualisation of the track. Both two-dimensional and three-dimensional representations were managed. The first one was simply implemented drawing in the output window three polylines representing the centreline and the two boundaries using a variable colour along the abscissa to have a visual dimensional reference; the second one was implemented using OpenGL graphic library. For the last representation a mapped quadrilateral mesh was generated sweeping the cross section of the track along curvilinear abscissa. To have a better definition in rendered view, trapezoidal cross kerbs were used. Three-dimensional model is very useful because permits to look the circuits from an arbitrary point of view even animating the visualisation linking the camera to the vehicle.

The trajectory was managed adopting the same data structure used for the track, defining an arbitrary number of control points that are located in the local track reference by the abscissa and centreline distance. In order to have a quick trajectory generation, an automated positioning algorithm for control points was developed; it locates control points at the start and at various inner positions of each segment depending on curvature and length of each segment. Furthermore automatic driving line generation takes advantage of the criteria used to place the control point for subsequent automatic driving line optimisation, being the nature of control point labelled (“straight”, “right bend”, “left bend”).

Three methods were considered to generate the trajectory from the control points: linear interpolation in local reference, cubic interpolation in local reference, cubic interpolation in global reference. The last method was adopted because it seems to generate the best results even if it can produce some crossings between track boundaries and trajectory.

The driving line is then represented by a simple vector of control points and its shape can be effectively controlled varying the distance from centreline of each control point.

RESULTS AND DISCUSSION – A practical application is described in this section to show the capability of proposed optimisation tool. The application concerns the tuning of a vehicle to get the best performance in a competition track.

The first parametric analysis regards the tuning of the engine. Torque progression is influenced by various set-up parameters, in this example the effect of reed valve material and exhaust pipe length was investigated. The engine parameters in standard condition are summarised in the table 1.

Table 1: Engine parameters.

Class	100 FA
Cooling	liquid
Bore	49.88 mm
Stroke	50.70 mm
Displacement	99.31 cm ³
Connecting rod length	100 mm
Crank case porting	reed valves
Ignition advance	2.3-2.4 mm
Max power	26 hp
Carburettor	φ24
Transmission ratio	10/78

Five configurations were considered varying each parameter per time. Adopted parameters are summarised in table 2 in which are also reported maximum torque and power obtained in simulations.

Table 2: Engine configurations.

	Standard	Type A	Type B	Type C	Type D
Reed valve (material/mm)	Carbon/0.25	Carbon/0.25	Carbon/0.25	Carbon/0.22	Glass/0.27
Exhaust length (mm)	485	480	490	485	485
Max power (kW@rpm)	19.5@12900	19.7@13000	19.4@12700	19.5@12700	19.1@12800
Max torque (Nm@rpm)	14.8@12200	15@12200	14.6@12100	15.2@11900	14.9@11600

Aerodynamic performances are then evaluated adopting the CFD approach. Surface pressure map and surface streamlines, obtained for a velocity of 90 km/h, are showed in figures 3a 3b.

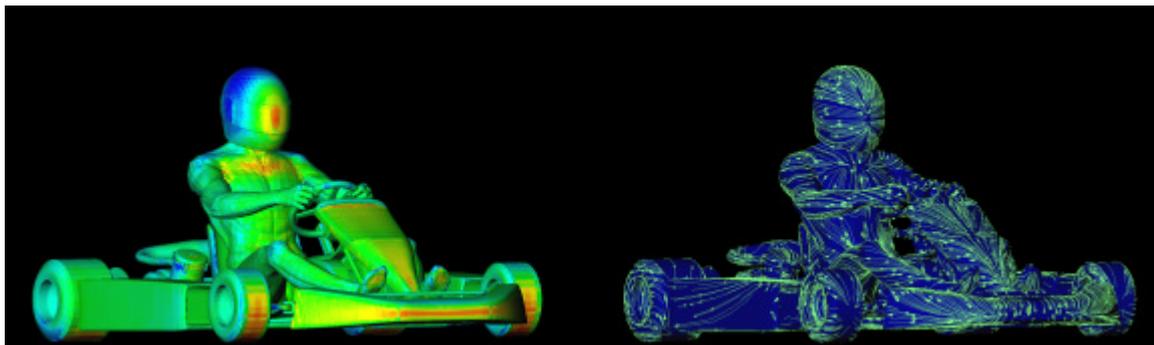


Figure 3. CFD results, speed 90km/h. a) Pressure map. b) Surface streamlines.

It can be noted the important contribution given by the driver to the frontal surface and to the velocity and pressure fields. Hence the attention was focused on the aerodynamic influence of the driver that, being directly exposed to the air flow, greatly modifies aerodynamic forces and moments. Different driver positions (shown in figure 4), including the absence of the driver, were analysed in order to estimate variations of aerodynamic forces and coefficients.



Figure 4. Aerodynamic simulations for two driver posture and no driver condition.

In table 3 the main results in terms of forces, moments and aerodynamic coefficients for the examined cases are collected and compared (see figure 2 for reference system adopted).

Table 3. Forces and aerodynamics coefficients for several kart configurations.

CASE	Velocity (km/h)	Mx (Nm)	My (Nm)	Mz (Nm)	CMx	CMy	CMz	V _a (N)	V _p (N)
Standard Driver	60	1.023	0.039	-1.261	0.01008	0.00038	-0.01242	11.33	-7.01
Standard Driver	90	2.812	-0.096	-3.180	0.01231	-0.00042	-0.01392	25.60	-15.41
Standard Driver	120	4.626	-0.332	-5.843	0.01139	-0.00082	-0.01439	45.62	-27.52
Hunched Driver	90	2.384	0.653	-1.972	0.01092	0.00299	-0.00903	21.79	-7.02
Absent Driver	90	0.441	1.120	3.847	0.00267	0.00677	0.02324	11.98	-20.29

Vehicle model was tuned considering lateral and longitudinal parameters. A typical g-g diagram for the herein considered kart is shown in figure 5. Experimental grip values were considered as the standard datum for presented investigation, furthermore a better (+10%) and a worst (-10%) conditions were accounted. The effect of transmission ratio was also considered modifying the base value (10/78) to get a faster (11/72) or slower (9/87) vehicle.

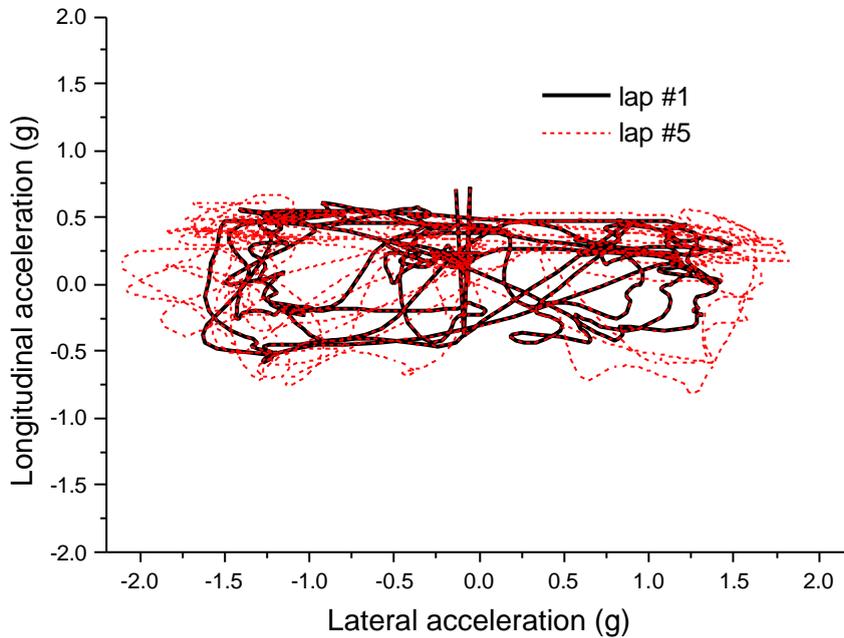


Figure 5: g-g diagram for two different complete laps

As far as the track model is concerned, the Italian track located near Parma was considered. Principal circuit data are summarised in table 4, acquired circuit map is represented in figure 6 in which a snapshot was taken from simulation tool graphic window. The three-dimensional representation of the circuit is represented in figure 7. The effect of several parameters was investigated. Obtained results are summarised in table 5; in figure 10 are reported the acceleration curves obtained for investigated configurations. Vehicle acceleration capability is the results of engine parameters and aerodynamic set-up but is not affected in our model by grip that influences only braking and cornering capability being the engine power easily transmitted to the road at each driving condition.

Table 4: Track parameters.

Length	1154 m
Width	8 m
Direction	Clockwise
Bends	9
Chicanes	3
Record Lap Time	45.033 s

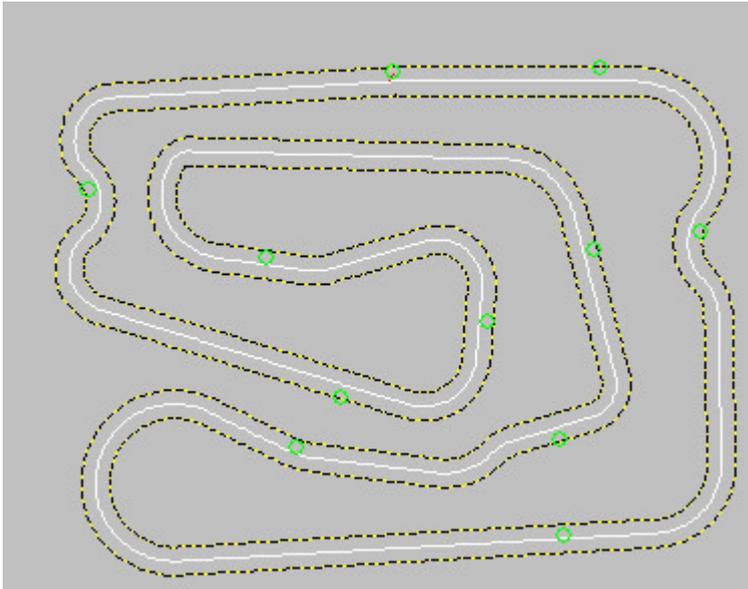


Figure 6. Parma circuit map.

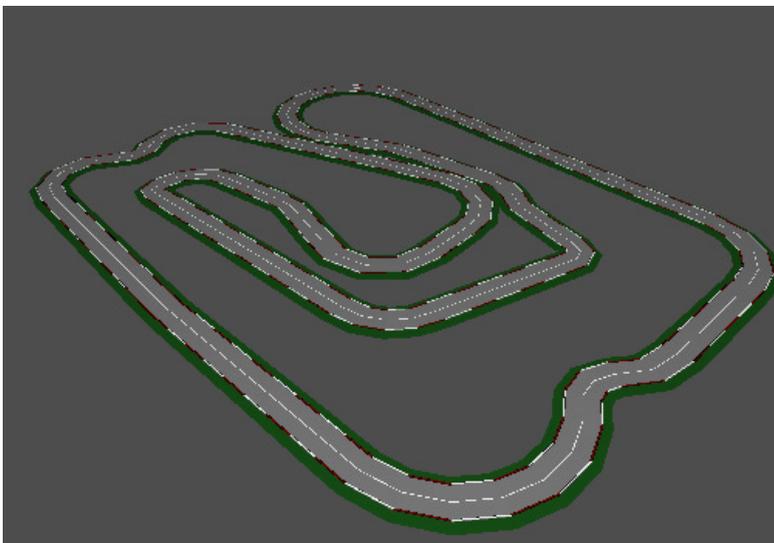


Figure 7. Parma circuit three dimensional visualisation.

The basic trajectory that interpolates the control points located at the centreline is exposed in figure 8 (Case 11 of table 5). Obviously this is not the best line but was considered to quantify the effect of line optimisation in lap time. In order to get the best line for the standard configuration of the vehicle, a series of optimisation steps were conducted. Subsequently the line was hand refined obtaining the minimum lap time for the path represented in figure 9 (Case 1 of table 5). Lap time is reduced of about five seconds optimising the line obtaining a

standard value of 45.31 s that is only at 0.3 s of track record. Starting from this solution the effect of each parameter was investigated. The improvement in aerodynamic penetration obtained with the hunched driver (Case 2 of table 5) gives 0.3 s. A strong negative effect is produced by the transmission ratio if lower values are adopted (Case 3), but a little margin of improvement is exhibited for lower values (Case 4) obtaining a reduction of 0.35 s in lap time.

Table 5: Integrated simulation results.

Case #	Description	Lap Time (s)	DT (s)
1	Standard	45.31	0.00
2	Cx hunched	45.03	-0.28
3	11/72	46.82	1.51
4	9/87	44.96	-0.35
5	exahust -5mm	45.34	0.03
6	exahust +5mm	45.32	0.01
7	reed 0.22 mm carbon fiber	45.31	0.00
8	reed 0.27 mm glass fiber	45.40	0.09
9	grip -10%	47.98	2.67
10	grip +10%	44.34	-0.97
11	centreline path	50.09	4.78

Engine set-up parameters investigated (Cases 5 to 8) produce only slight variations in resulting lap time and the standard configuration seems to be the best for this track. As far as the grip is concerned a strong effect is observed with a consistent decay obtained with the worst adherence (2.67 s, Case 9) and a good improvement for the best adherence (-.97, Case 10).

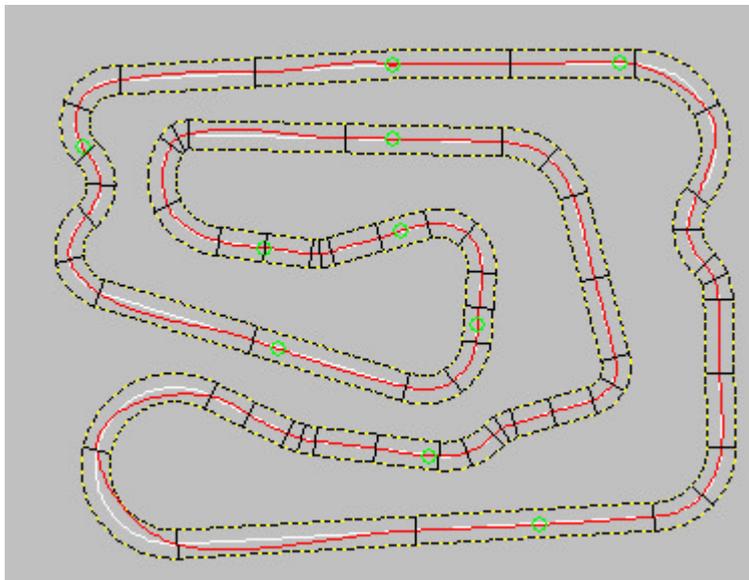


Figure 8. Centreline trajectory.

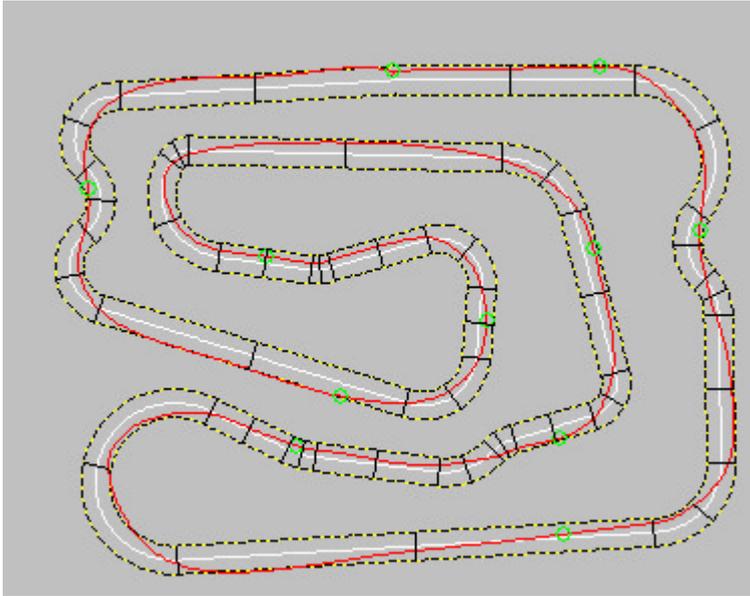


Figure 9: Optimised line.

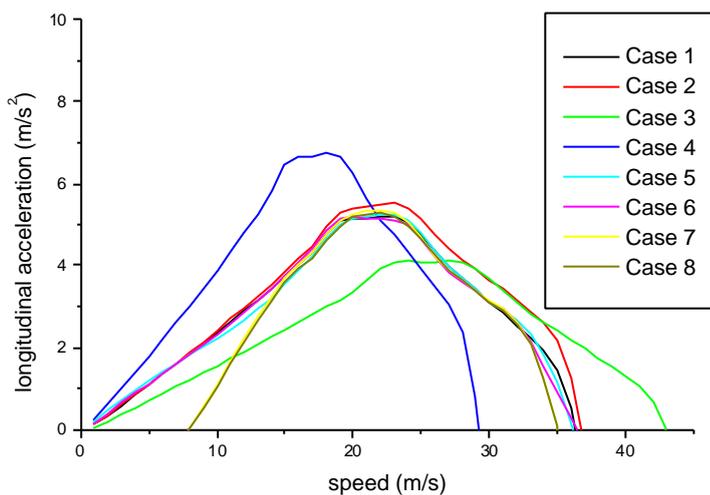


Figure 10: Vehicle acceleration capability for investigated configurations.

Presented analysis is very useful to quantify the effect of each parameters involved. Obviously better performance could be achieved with simultaneous variations of all parameters. For instance, to take the full advantage of a different torque progression, also the transmission ratio has to be optimised. Furthermore, optimum line depend from the relative values of cornering and driving/braking capability, so each configuration requires a further line optimisation. At this stage an integrated optimisation algorithm is still missing, but it will easily implemented in future version of the code.

CONCLUSIONS – In this paper an integrated simulation tool for lap time analysis was presented. The attention was focused on competition go karts considering the peculiar features of such vehicles. Aerodynamic penetration was investigated by means of CFD analysis that allows to evaluate the effect of driver position on global drag forces. Engine was modelled by means of an integrated one-dimensional gasdynamic model. Vehicle was modelled with a complex non linear simulator including the effects of frame set-up and tires

behaviour. Tracks were represented with a general purpose model able to manage racing line in competition circuit and in testing tracks.

Sub models information were collected in a simulation tool to perform lap time analysis considering the effect of each parameter.

A practical application was presented. Computed lap times obtained are consistent with the track record and the effect of various parameters was investigated and discussed.

Further improvement of the proposed tool are going to be introduced including an automated driver suitable to perform the lap time controlling the non linear model during the simulation, and an optimisation algorithm capable to control all design and set-up parameters simultaneously [18].

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